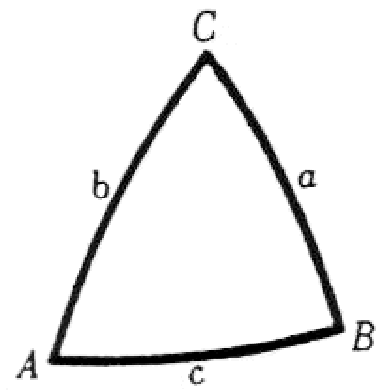
FORMULAS DE TRIGONOMETRIA ESFERICA

Ley de los senos



I)
$$\begin{aligned} \sin c \sin A &= \sin a \sin C \\ \sin a \sin B &= \sin b \sin A \\ \sin b \sin C &= \sin c \sin B \end{aligned}$$

Ley de los cosenos para lados

II)
$$\begin{aligned} \cos a &= \cos b \cos c + \sin b \sin c \cos A \\ \cos b &= \cos c \cos a + \sin c \sin a \cos B \\ \cos c &= \cos a \cos b + \sin a \sin b \cos C \end{aligned}$$

Ley de los cosenos para ángulos

III)
$$\begin{aligned} \cos A &= -\cos B \cos C + \sin B \sin C \cos a \\ \cos B &= -\cos C \cos A + \sin C \sin A \cos b \\ \cos C &= -\cos A \cos B + \sin A \sin B \cos c \end{aligned}$$

IV)
$$\begin{aligned} \sin a \cos B &= \cos b \sin c - \sin b \cos c \cos A \\ \sin a \cos C &= \cos c \sin b - \sin c \cos b \cos A \\ \sin b \cos A &= \cos a \sin c - \sin a \cos c \cos B \\ \sin b \cos C &= \cos c \sin a - \sin c \cos a \cos B \\ \sin c \cos A &= \cos a \sin b - \sin a \cos b \cos C \\ \sin c \cos B &= \cos b \sin a - \sin b \cos a \cos C \end{aligned}$$

V)
$$\begin{aligned} \sin A \cos b &= \cos B \sin C + \cos C \sin B \cos a \\ \sin A \cos c &= \cos C \sin B + \cos B \sin C \cos a \\ \sin B \cos a &= \cos A \sin C + \cos C \sin A \cos b \\ \sin B \cos c &= \cos C \sin A + \cos A \sin C \cos b \\ \sin C \cos a &= \cos A \sin B + \cos B \sin A \cos c \\ \sin C \cos b &= \cos B \sin A + \cos A \sin B \cos c \end{aligned}$$

VI)

$$\sin a \cot b = \cot B \sin C + \cos C \cos a$$
$$\sin a \cot c = \cot C \sin B + \cos B \cos a$$
$$\sin b \cot a = \cot A \sin C + \cos C \cos b$$
$$\sin b \cot c = \cot C \sin A + \cos A \cos b$$
$$\sin c \cot a = \cot A \sin B + \cos B \cos c$$
$$\sin c \cot b = \cot B \sin A + \cos A \cos c$$

VII)

$$\sin A \cot B = \cot b \sin c - \cos c \cos A$$
$$\sin A \cot C = \cot c \sin b - \cos b \cos A$$
$$\sin B \cot A = \cot a \sin c - \cos c \cos B$$
$$\sin B \cot C = \cot c \sin a - \cos a \cos B$$
$$\sin C \cot A = \cot a \sin b - \cos b \cos C$$
$$\sin C \cot B = \cot b \sin a - \cos a \cos C$$

Ley de las tangentes

VIII)

$$\frac{\tan\frac{(A-B)}{2}}{\tan\frac{(A+B)}{2}} = \frac{\tan\frac{(a-b)}{2}}{\tan\frac{(a+b)}{2}} \qquad \frac{\tan\frac{(A-C)}{2}}{\tan\frac{(A+C)}{2}} = \frac{\tan\frac{(a-c)}{2}}{\tan\frac{(a+c)}{2}}$$

$$\frac{\tan\frac{(B-C)}{2}}{\tan\frac{(B+C)}{2}} = \frac{\tan\frac{(b-c)}{2}}{\tan\frac{(b+c)}{2}}$$

Fórmulas de los semiángulos

$$s = \frac{a+b+c}{2}$$

$$\sin^2\frac{A}{2} = \frac{\sin(s-b)\sin(s-c)}{\sin b \sin c} \qquad \sin^2\frac{B}{2} = \frac{\sin(s-c)\sin(s-a)}{\sin a \sin c}$$

IX)

$$\sin^2\frac{C}{2} = \frac{\sin(s-a)\sin(s-b)}{\sin a \sin b}$$

$$\cos^2\frac{A}{2} = \frac{\sin(s)\sin(s-a)}{\sin b \sin c} \qquad \cos^2\frac{B}{2} = \frac{\sin(s)\sin(s-b)}{\sin a \sin c}$$

X)

$$\cos^2\frac{C}{2} = \frac{\sin(s)\sin(s-c)}{\sin a \sin b}$$

$$\tan^2\frac{A}{2} = \frac{\sin(s-b)\sin(s-c)}{\sin(s)\sin(s-a)} \qquad \tan^2\frac{B}{2} = \frac{\sin(s-c)\sin(s-a)}{\sin s \sin(s-b)}$$

XI)

$$\tan^2\frac{C}{2} = \frac{\sin(s-a)\sin(s-b)}{\sin(s)\sin(s-c)}$$

Fórmulas de los semilados

$$S = \frac{A + B + C}{2}$$

XII)

$$\sin^2 \frac{a}{2} = \frac{\cos(S)\cos(S - A)}{\sin B \sin C} \qquad \sin^2 \frac{b}{2} = \frac{\cos(S)\cos(S - B)}{\sin A \sin C}$$

$$\sin^2 \frac{c}{2} = \frac{\cos(S)\cos(S - C)}{\sin A \sin B}$$

XIII)

$$\cos^2 \frac{a}{2} = \frac{\cos(S - B)\cos(S - C)}{\sin B \sin C} \qquad \cos^2 \frac{b}{2} = \frac{\cos(S - A)\cos(S - C)}{\sin A \sin C}$$

$$\cos^2 \frac{c}{2} = \frac{\cos(S - A)\cos(S - B)}{\sin a \sin b}$$

XIV)

$$\tan^2 \frac{a}{2} = \frac{\cos(S)\cos(S - A)}{\cos(S - B)\cos(S - C)} \qquad \tan^2 \frac{b}{2} = \frac{\cos(S)\cos(S - B)}{\cos(S - A)\sin(S - C)}$$

$$\tan^2 \frac{c}{2} = \frac{\cos(S)\cos(S - C)}{\cos(S - A)\cos(S - B)}$$

Analogías de Neper

XV)

$$\frac{\sin\frac{(A-B)}{2}}{\sin\frac{(A+B)}{2}}=\frac{\tan\frac{(a-b)}{2}}{\tan\frac{c}{2}} \qquad \frac{\sin\frac{(a-b)}{2}}{\sin\frac{(a+b)}{2}}=\frac{\tan\frac{(A-B)}{2}}{\tan\frac{C}{2}}$$

$$\frac{\cos\frac{(A-B)}{2}}{\cos\frac{(A+B)}{2}}=\frac{\tan\frac{(a+b)}{2}}{\tan\frac{c}{2}} \qquad \frac{\cos\frac{(a-b)}{2}}{\cos\frac{(a+b)}{2}}=\frac{\tan\frac{(A+B)}{2}}{\tan\frac{C}{2}}$$

XVI)

$$\frac{\sin\frac{(A-C)}{2}}{\sin\frac{(A+C)}{2}}=\frac{\tan\frac{(a-c)}{2}}{\tan\frac{b}{2}} \qquad \frac{\sin\frac{(a-c)}{2}}{\sin\frac{(a+c)}{2}}=\frac{\tan\frac{(A-C)}{2}}{\tan\frac{B}{2}}$$

$$\frac{\cos\frac{(A-C)}{2}}{\cos\frac{(A+C)}{2}}=\frac{\tan\frac{(a+c)}{2}}{\tan\frac{b}{2}} \qquad \frac{\cos\frac{(a-c)}{2}}{\cos\frac{(a+c)}{2}}=\frac{\tan\frac{(A+C)}{2}}{\tan\frac{B}{2}}$$

XVII)

$$\frac{\sin\frac{(B-C)}{2}}{\sin\frac{(B+C)}{2}}=\frac{\tan\frac{(b-c)}{2}}{\tan\frac{a}{2}} \qquad \frac{\sin\frac{(b-c)}{2}}{\sin\frac{(b+c)}{2}}=\frac{\tan\frac{(B-C)}{2}}{\tan\frac{A}{2}}$$

$$\frac{\cos\frac{(B-C)}{2}}{\cos\frac{(B+C)}{2}}=\frac{\tan\frac{(b+c)}{2}}{\tan\frac{a}{2}} \qquad \frac{\cos\frac{(b-c)}{2}}{\cos\frac{(b+c)}{2}}=\frac{\tan\frac{(B+C)}{2}}{\tan\frac{A}{2}}$$

Fórmulas de Gauss

XVIII)

$$\frac{\sin\frac{(a-b)}{2}}{\sin\frac{c}{2}} = \frac{\sin\frac{(A-B)}{2}}{\cos\frac{C}{2}} \qquad \frac{\sin\frac{(a+b)}{2}}{\sin\frac{c}{2}} = \frac{\cos\frac{(A-B)}{2}}{\sin\frac{C}{2}}$$

$$\frac{\cos\frac{(a-b)}{2}}{\cos\frac{c}{2}} = \frac{\sin\frac{(A+B)}{2}}{\cos\frac{C}{2}} \qquad \frac{\cos\frac{(a+b)}{2}}{\cos\frac{c}{2}} = \frac{\cos\frac{(A+B)}{2}}{\sin\frac{C}{2}}$$

XIX)

$$\frac{\sin\frac{(a-c)}{2}}{\sin\frac{b}{2}} = \frac{\sin\frac{(A-C)}{2}}{\cos\frac{B}{2}} \qquad \frac{\sin\frac{(a+c)}{2}}{\sin\frac{b}{2}} = \frac{\cos\frac{(A-C)}{2}}{\sin\frac{B}{2}}$$

$$\frac{\cos\frac{(a-c)}{2}}{\cos\frac{b}{2}} = \frac{\sin\frac{(A+B)}{2}}{\cos\frac{B}{2}} \qquad \frac{\cos\frac{(a+c)}{2}}{\cos\frac{b}{2}} = \frac{\cos\frac{(A+C)}{2}}{\sin\frac{B}{2}}$$

XX)

$$\frac{\sin\frac{(b-c)}{2}}{\sin\frac{a}{2}} = \frac{\sin\frac{(B-C)}{2}}{\cos\frac{A}{2}} \qquad \frac{\sin\frac{(b+c)}{2}}{\sin\frac{a}{2}} = \frac{\cos\frac{(B-C)}{2}}{\sin\frac{A}{2}}$$

$$\frac{\cos\frac{(b-c)}{2}}{\cos\frac{a}{2}} = \frac{\sin\frac{(B+C)}{2}}{\cos\frac{A}{2}} \qquad \frac{\cos\frac{(b+c)}{2}}{\cos\frac{a}{2}} = \frac{\cos\frac{(B+C)}{2}}{\sin\frac{A}{2}}$$